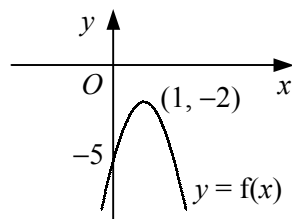


- 1 Sketch and label each pair of graphs on the same set of axes showing the coordinates of any points where the graphs intersect. Write down the equations of any asymptotes.
- a** $y = x^2$ and $y = x^3$ **b** $y = x^2$ and $y = x^4$
c $y = \frac{1}{x}$ and $y = \frac{1}{x^2}$ **d** $y = x$ and $y = \sqrt{x}$
e $y = x^2$ and $y = 3x^2$ **f** $y = \frac{1}{x}$ and $y = \frac{2}{x}$
- 2 $f(x) = (x - 1)(x - 3)(x - 4)$.
- a** Find $f(0)$.
b Write down the solutions of the equation $f(x) = 0$.
c Sketch the curve $y = f(x)$.
- 3 Sketch each graph showing the coordinates of any points of intersection with the coordinate axes.
- a** $y = (x + 1)(x - 1)(x - 3)$ **b** $y = 2x(x - 1)(x - 5)$
c $y = -(x + 2)(x + 1)(x - 2)$ **d** $y = x^2(x - 4)$
e $y = 3x(2 + x)(1 - x)$ **f** $y = (x + 2)(x - 1)^2$
- 4 **a** Factorise fully $x^3 + 6x^2 + 9x$.
b Hence, sketch the curve $y = x^3 + 6x^2 + 9x$, showing the coordinates of any points where the curve meets the coordinate axes.
- 5 Given that the constants p and q are such that $p > q > 0$, sketch each of the following graphs showing the coordinates of any points of intersection with the coordinate axes.
- a** $y = (x - p)(x - q)^2$ **b** $y = (x - p)(x^2 - q^2)$

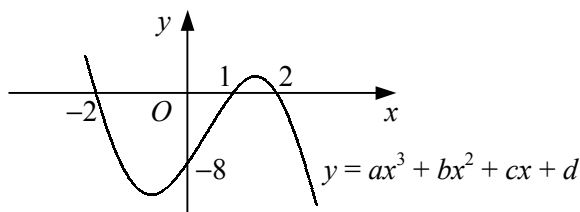
6



The diagram shows the curve with equation $y = f(x)$ which has a turning point at $(1, -2)$ and crosses the y -axis at the point $(0, -5)$.

Given that $f(x)$ is a quadratic function, find an expression for $f(x)$.

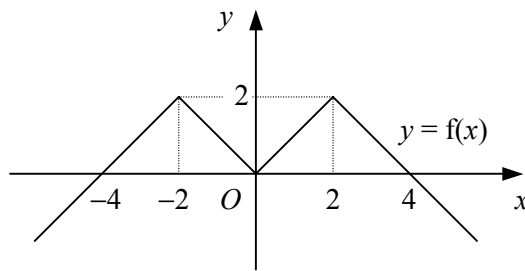
7



The diagram shows the curve with equation $y = ax^3 + bx^2 + cx + d$.

Given that the curve crosses the y -axis at the point $(0, -8)$ and crosses the x -axis at the points $(-2, 0)$, $(1, 0)$ and $(2, 0)$, find the values of the constants a , b , c and d .

8



The diagram shows the graph of $y = f(x)$.

Use the graph to write down the number of solutions that exist to each of the following equations.

- a** $f(x) = 1$ **b** $f(x) = 3$ **c** $f(x) = -1$ **d** $f(x) = 0$

- 9** **a** Sketch on the same set of axes the graphs of $y = x^2$ and $y = 1 - 2x$.
b Hence state the number of roots that the equation $x^2 + 2x - 1 = 0$ has and give a reason for your answer.
- 10** **a** Find the coordinates of the vertex of the curve $y = x^2 + 2x - 3$.
b By sketching two suitable graphs on the same set of axes, show that the equation
- $$x^2 + 2x - 3 - \frac{1}{x} = 0$$
- has one positive and two negative real roots.

- 11** Show that the line $y = x - 3$ is a tangent to the curve $y = x^2 - 5x + 6$.

- 12** **a** Solve the simultaneous equations

$$y = 3x + 7$$

$$y = x^2 + 5x + 8$$

- b** Hence, describe the geometrical relationship between the straight line $y = 3x + 7$ and the curve $y = x^2 + 5x + 8$.

- 13** **a** Find the coordinates of the points where the straight line $y = x + 6$ meets the curve $y = x^3 - 4x^2 + x + 6$.

- b** Given that

$$x^3 - 4x^2 + x + 6 \equiv (x + 1)(x - 2)(x - 3),$$

sketch the straight line $y = x + 6$ and the curve $y = x^3 - 4x^2 + x + 6$ on the same diagram, showing the coordinates of the points where the curve crosses the coordinate axes.

- 14** Find the value of the constant k such that the straight line with equation $y = 3x + k$ is a tangent to the curve with equation $y = 2x^2 - 5x + 1$.

- 15** Find the set of values of the constant a for which the line $y = 2 - 5x$ intersects the curve $y = x^2 + ax + 18$ at two points.

- 16** The curve C has the equation $y = x^2 - 2x + 6$.

- a** Find the values of p for which the line $y = px + p$ is a tangent to the curve C .
b Prove that there are no real values of q for which the line $y = qx + 7$ is a tangent to the curve C .

1 Describe how the graph of $y = f(x)$ is transformed to give the graph of

a $y = f(x - 1)$

b $y = f(x) - 3$

c $y = 2f(x)$

d $y = f(4x)$

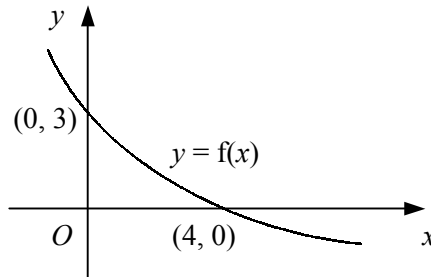
e $y = -f(x)$

f $y = \frac{1}{5}f(x)$

g $y = f(-x)$

h $y = f(\frac{2}{3}x)$

2



The diagram shows the curve with equation $y = f(x)$ which crosses the coordinate axes at the points $(0, 3)$ and $(4, 0)$.

Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of

a $y = 3f(x)$

b $y = f(x + 4)$

c $y = -f(x)$

d $y = f(\frac{1}{2}x)$

3 Find and simplify an equation of the graph obtained when

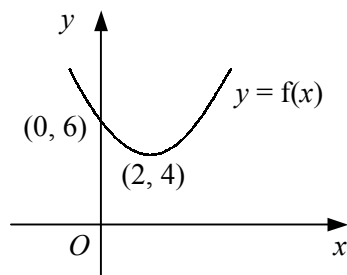
a the graph of $y = 2x + 5$ is translated by 1 unit in the positive y -direction,

b the graph of $y = 1 - 4x$ is stretched by a factor of 3 in the y -direction, about the x -axis,

c the graph of $y = 3x + 1$ is translated by 4 units in the negative x -direction,

d the graph of $y = 4x - 7$ is reflected in the x -axis.

4



The diagram shows the curve with equation $y = f(x)$ which has a turning point at $(2, 4)$ and crosses the y -axis at the point $(0, 6)$.

Showing the coordinates of the turning point and of any points of intersection with the axes, sketch on separate diagrams the graphs of

a $y = f(x) - 3$

b $y = f(x + 2)$

c $y = f(2x)$

d $y = \frac{1}{2}f(x)$

5 Describe a single transformation that would map the graph of $y = x^3$ onto the graph of

a $y = 4x^3$

b $y = (x - 2)^3$

c $y = -x^3$

d $y = x^3 + 5$

6 Describe a single transformation that would map the graph of $y = x^2 + 2$ onto the graph of

a $y = 2x^2 + 4$

b $y = x^2 - 5$

c $y = \frac{1}{9}x^2 + 2$

d $y = x^2 + 4x + 6$

- 7 Find and simplify an equation of the graph obtained when
- the graph of $y = x^2 + 2x$ is translated by 1 unit in the positive x -direction,
 - the graph of $y = x^2 - 4x + 5$ is stretched by a factor of $\frac{1}{3}$ in the x -direction, about the y -axis.
 - the graph of $y = x^2 + x - 6$ is reflected in the y -axis,
 - the graph of $y = 2x^2 - 3x$ is stretched by a factor of 2 in the x -direction, about the y -axis.

8 $f(x) \equiv x^2 - 4x$.

- Find the coordinates of the vertex of the graph $y = f(x)$.
- Sketch each pair of graphs on the same set of axes showing the coordinates of the vertex of each graph.
 - $y = f(x)$ and $y = 3 + f(x)$
 - $y = f(x)$ and $y = f(x - 2)$
 - $y = f(x)$ and $y = f(2x)$

- 9 Sketch each pair of graphs on the same set of axes.

- $y = x^2$ and $y = (x + 3)^2$
- $y = x^3$ and $y = x^3 + 4$
- $y = \frac{1}{x}$ and $y = \frac{1}{x-2}$
- $y = \sqrt{x}$ and $y = \sqrt{2x}$

- 10
 - Describe two different transformations, each of which would map the graph of $y = \frac{1}{x}$ onto the graph of $y = \frac{1}{3x}$.
 - Describe two different transformations, each of which would map the graph of $y = x^2$ onto the graph of $y = 4x^2$.

11 $f(x) \equiv (x + 4)(x + 2)(x - 1)$.

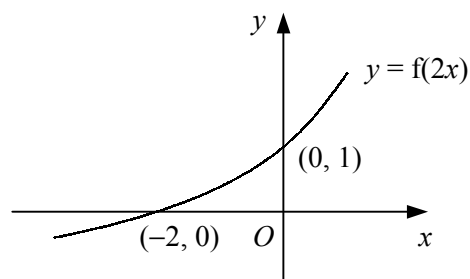
Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of

- $y = f(x)$
- $y = f(x - 4)$
- $y = f(-x)$
- $y = f(2x)$

- 12 The curve $y = f(x)$ is a parabola and the coordinates of its vertex are (a, b) . Write down, in terms of a and b , the coordinates of the vertex of the graph

- $y = 3f(x)$
- $y = 4 + f(x)$
- $y = f(x + 1)$
- $y = f(\frac{1}{3}x)$

13



The diagram shows the curve with equation $y = f(2x)$ which crosses the coordinate axes at the points $(-2, 0)$ and $(0, 1)$.

Showing the coordinates of any points of intersection with the coordinate axes, sketch on separate diagrams the curves

- $y = 3f(2x)$
- $y = f(x)$

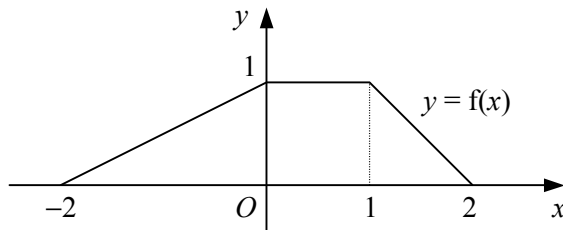
- 1 a Solve the simultaneous equations

$$y = 3x - 4$$

$$y = 4x^2 - 9x + 5 \quad (4)$$

- b Hence, describe the geometrical relationship between the straight line $y = 3x - 4$ and the curve $y = 4x^2 - 9x + 5$. (1)

2



The diagram shows the graph of $y = f(x)$ which is defined for $-2 \leq x \leq 2$.

Labelling the axes in a similar way, sketch on separate diagrams the graphs of

a $y = 3f(x)$, (2)

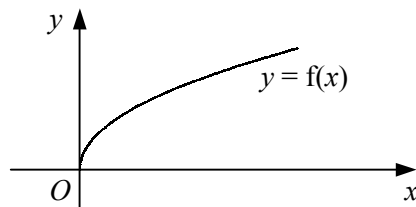
b $y = f(x + 1)$, (2)

c $y = f(-x)$. (2)

- 3 a Show that the line $y = 4x + 1$ does not intersect the curve $y = x^2 + 5x + 2$. (4)

- b Find the values of m such that the line $y = mx + 1$ meets the curve $y = x^2 + 5x + 2$ at exactly one point. (4)

4



The diagram shows the curve with the equation $y = f(x)$ where

$$f(x) \equiv \sqrt{x}, \quad x \geq 0.$$

- a Sketch on the same set of axes the graphs of $y = 1 + f(x)$ and $y = f(x + 3)$. (4)

- b Find the coordinates of the point of intersection of the two graphs drawn in part a. (4)

- 5 The curve C has the equation $y = x^2 + kx - 3$ and the line l has the equation $y = k - x$, where k is a constant.

Prove that for all real values of k , the line l will intersect the curve C at exactly two points. (7)

6

$$f(x) \equiv 2x^2 - 4x + 5.$$

- a Express $f(x)$ in the form $a(x + b)^2 + c$. (3)

- b Showing the coordinates of the vertex in each case, sketch on the same set of axes the curves

i $y = f(x)$,

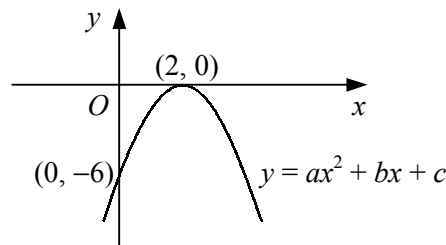
ii $y = f(x + 3)$. (4)

- 7 a Sketch on the same diagram the straight line $y = 2x - 5$ and the curve $y = x^3 - 3x^2$, showing the coordinates of any points where each graph meets the coordinate axes. (4)
- b Hence, state the number of real roots that exist for the equation

$$x^3 - 3x^2 - 2x + 5 = 0,$$

giving a reason for your answer. (2)

8



The diagram shows the curve with the equation $y = ax^2 + bx + c$.

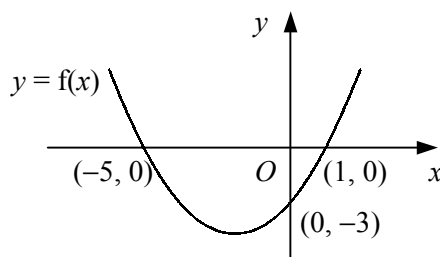
Given that the curve crosses the y -axis at the point $(0, -6)$ and touches the x -axis at the point $(2, 0)$, find the values of the constants a , b and c . (6)

- 9 a Show that

$$(1 - x)(2 + x)^2 \equiv 4 - 3x^2 - x^3. \quad (3)$$

- b Hence, sketch the curve $y = 4 - 3x^2 - x^3$, showing the coordinates of any points of intersection with the coordinate axes. (3)

10



The diagram shows the curve with equation $y = f(x)$ which crosses the coordinate axes at the points $(-5, 0)$, $(1, 0)$ and $(0, -3)$.

Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the curves

a $y = -f(x)$, (2)

b $y = f(x - 5)$, (2)

c $y = f(2x)$. (2)

- 11 a Describe fully the transformation that maps the graph of $y = f(x)$ onto the graph of $y = f(x + 1)$. (2)

- b Sketch the graph of $y = \frac{1}{x+1}$, showing the coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes. (3)

- c By sketching another suitable curve on your diagram in part b, show that the equation

$$x^3 - \frac{1}{x+1} = 2$$

has one positive and one negative real root. (4)